Lecture 11

Thursday, January 27, 2022 8:33 PM

* Prayer
* Spiritual thought

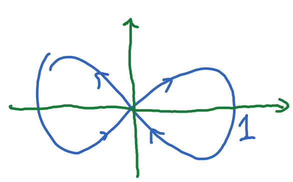
the acceleration is, in general, not tangent to the trajectory. Think of driving along a circular path.

$$a(t) = r''(t) = a_T T + a_N N$$

$a_T = V'(t)$ - where $V(t)$ is the speed (not velocity)

$a_N = \kappa V(t)^2$ - where κ is the curvature.

Ex:



$\begin{cases} x = \sin t \\ y = \sin 2t \end{cases}$ $\quad 0 \le t \le 2\pi$

Driving along a figure 8. Find the tangent/normal acceleration at (1,0).

(1,0) corresponds to $t = \frac{\pi}{2}$.

$$r(t) = \langle \sin t, \sin 2t, 0 \rangle$$

$$r'(t) = \langle \cos t, -2\cos 2t, 0 \rangle$$

$$V(t) = |r'(t)| = \sqrt{\cos^2 t + 4\cos^2 2t} \rightsquigarrow a_T = V'(t) = \frac{-2\sin t\cos t - 16\cos 2t \sin 2t}{2\sqrt{\cos^2 t + 4\cos^2 2t}}$$

At $t = \frac{\pi}{2}$, $a_T = 0$.

$$\boxed{a_T = \frac{r(t) \cdot r''(t)}{|r'(t)|} \qquad a_N = \frac{|r'(t) \times r''(t)|}{|r'(t)|}}$$

$r'(t) = \langle \cos t, -2\cos 2t, 0 \rangle$

$r''(t) = \langle -\sin t, 4\sin 2t, 0 \rangle$

$r'(t) \times r''(t) = \langle 0, 0, 4\cos t \sin 2t - 2\sin t \cos 2t \rangle$

At $t = \frac{\pi}{2}$: $r'(t) \times r''(t) = \langle 0, 0, 2 \rangle$

$|r'(t)| = |\langle -1, 0, 0 \rangle| = 1$

Thus, $a_N = 2$.

Know velocity and initial position $\rightarrow$ know all positions

Ex: $v(t) = r'(t) = \langle 1, t, t^2 \rangle$

$r(0) = \langle 2, 1, 0 \rangle$

$$r(t) - r(0) = \int_0^t r'(s)\,ds = \int_0^t \langle 1, s, s^2 \rangle\,ds = \left\langle s, \frac{s^2}{2}, \frac{s^3}{3} \right\rangle \Big|_0^t$$

$$= \left\langle t, \frac{t^2}{2}, \frac{t^3}{3} \right\rangle$$

$$r(t) = r(0) + \left\langle t, \frac{t^2}{2}, \frac{t^3}{3} \right\rangle = \left\langle 2 + t, 1 + \frac{t^2}{2}, \frac{t^3}{3} \right\rangle.$$

Functions of more than one variable:

$f(x, y, \ldots)$

Domain of f = the set of points $(x, y, \ldots)$ where f is defined.

C-Level set of f = set of points $(x, y, \ldots)$ such that $f(x, y, \ldots) = C$.

Ex $\quad f(x,y) = \sqrt{1 - x^2 + y^2}\ \ln(1+y)$

Domain: $\{(x,y) : 1 - x^2 + y^2 \geq 0,\ 1 + y > 0\}$

$= \{(x,y) : x^2 - y^2 \leq 1,\ y > -1\}$

On Mathematica:

RegionPlot[x^2 − y^2 ≤ 1&&y > −1, {x, −5,5}, {y, −5,5}]

* Level set:

1-level set : $\{(x,y) : f(x,y) = 1\}$

2-level set : $\{(x,y) : f(x,y) = 2\}$

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On Mathematica:

ContourPlot[f[x, y] == 1, {x, −3,3}, {y, −1,3}]

* Graph:

$\{(x,y,z) : z = f(x,y)\}$